

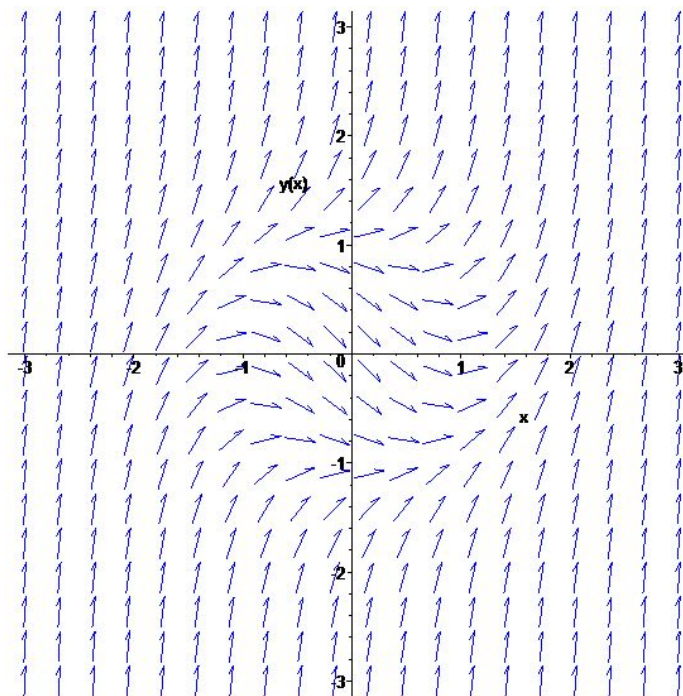
Section 6.2. Direction Fields

For first-order DEs (but *only* for first-order DEs), we can get a qualitative understanding of solution curves from a direction field (or slope field).

Definition: For a differential equation $dy/dx = g(x, y)$ the **direction field** (or slope field) is formed by drawing at every point (x_0, y_0) in the xy -plane a small vector of slope $g(x_0, y_0)$.

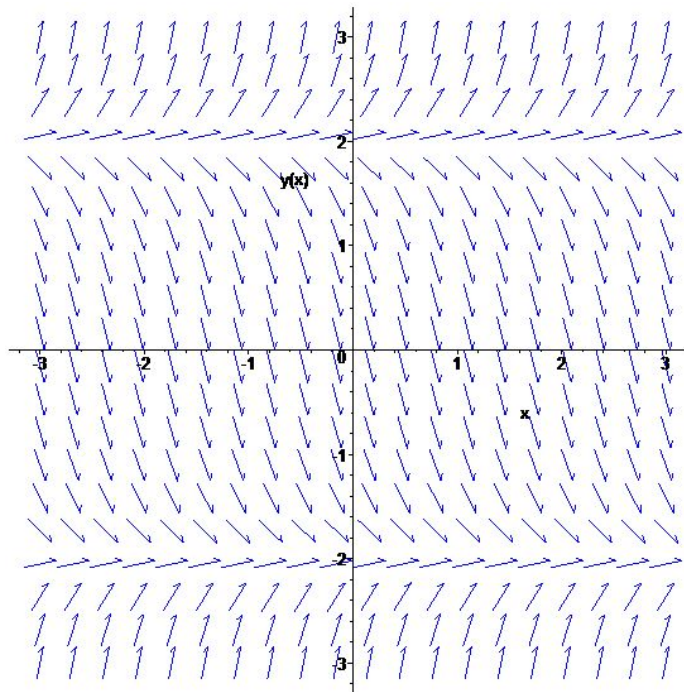
Direction Fields

Example: $dy/dx = x^2 + y^2 - 1$



Direction Fields

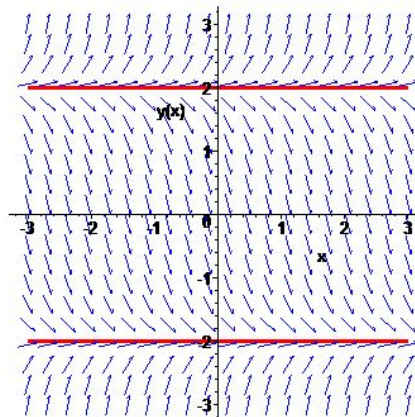
Example: $dy/dx = y^2 - 4$



Section 6.3. Equilibria, Stability, and Phase Plots

Definition: An **equilibrium solution** of an DE is a constant solution $y = C$. The graph of any equilibrium solution in the xy -plane is a horizontal line.

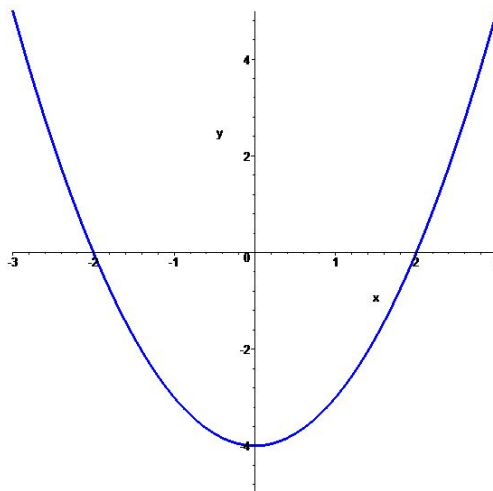
Example: The equilibrium solutions of $dy/dx = y^2 - 4$ are $y = 2$ and $y = -2$.



Equilibria, Stability, and Phase Plots

Definition: A **phase plot** of an DE $dy/dx = g(y)$ is a plot of y' vs. y . (A phase plot is a tool for understanding autonomous, first-order DEs.)

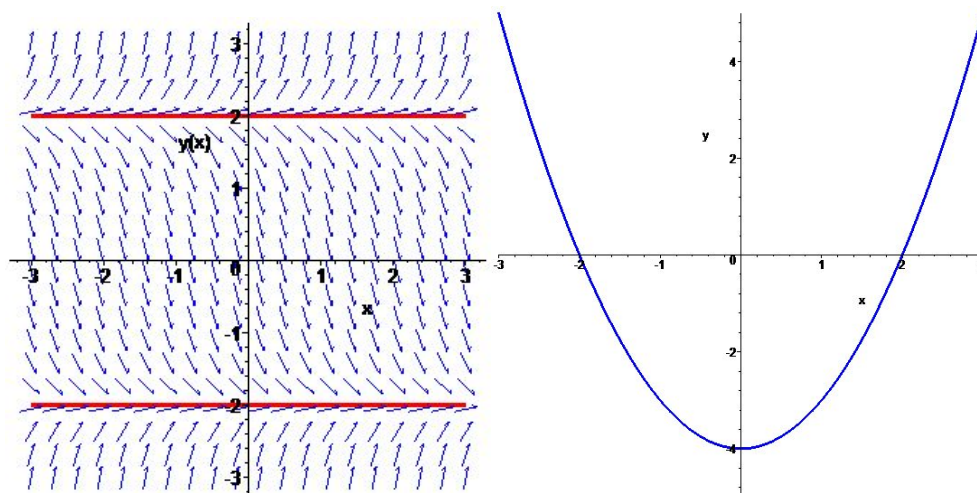
Example: The phase plot for $dy/dx = y^2 - 4$ is:



Equilibria, Stability, and Phase Plots

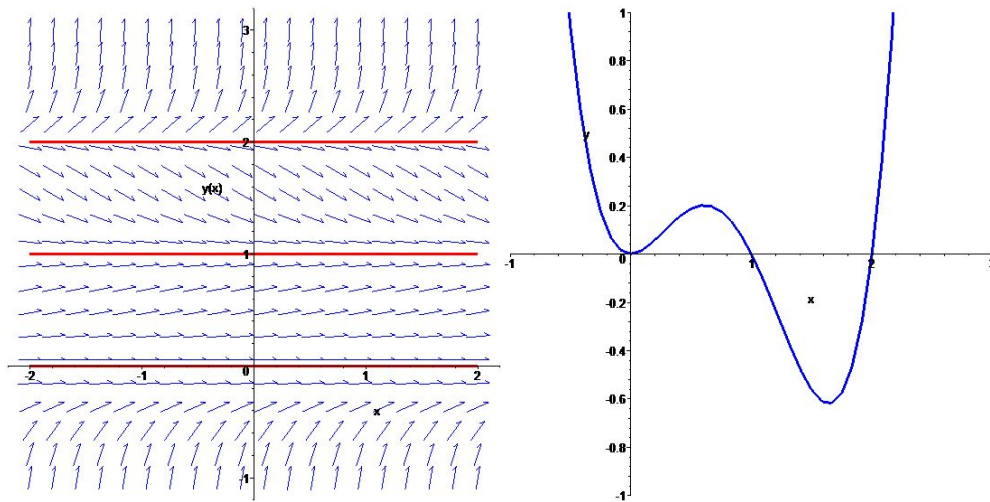
Phase plots can be used to predict the *stability* of equilibrium solutions.

Example: $dy/dx = y^2 - 4$



Equilibria, Stability, and Phase Plots

Example: $dy/dx = y^4 - 3y^3 + 2y^2$



Equilibria, Stability, and Phase Plots

Example: Find all equilibrium solutions to the ODE

$$y' = 10y - 0.2y^2$$

and determine the stability of each by using a phase plot.